

Nuclear masses, deformations and shell effects

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Abstract. We show that the Liquid Drop Model is best suited to describe the masses of prolate deformed nuclei than of spherical nuclei. To this end three Liquid Drop Mass formulas are employed to describe nuclear masses of eight sets of nuclei with similar quadrupole deformations. It is shown that they are able to fit the measured masses of prolate deformed nuclei with an RMS smaller than 750 keV, while for the spherical nuclei the RMS is, in the three cases, larger than 2000 keV. The RMS of the best fit of the masses of semi-magic nuclei is also larger than 2000 keV. The parameters of the three models are studied, showing that the surface symmetry term is the one which varies the most from one group of nuclei to another. In one model, isospin dependent terms are also found to exhibit strong changes. The inclusion of shell effects allows for better fits, which continue to be better in the prolate deformed nuclei region.

1. Introduction

The description of nuclear masses in terms of the Liquid Drop Model paved the way to the basic understanding of nuclear properties, like the saturation of the nuclear force, the existence of pairing and shell effects, and the description of fission and fusion processes [1]. The Q-values of different nuclear reactions, obtained from mass differences, must be accurately known to allow the description of the astrophysical origin of the elements [2]. Accurate theoretical predictions of nuclear masses remain a challenge [3], sharing the difficulties with other quantum many-body calculations, and complicated by the absence of a full theory of the nuclear interaction.

Decades of work have produced microscopic and macroscopic mass formulas [4]. At present, the most successful approaches seem to be the Finite Range Droplet Model (FRDM) [5], the Skyrme and Gogny Hartree Fock Bogolyubov (HFB) [6, 7], and the Duflo-Zuker (DZ) mass formula [8, 9, 10]. They allow for the calculation of masses, charge radii, deformations, and in some cases also fission barriers. They all contain a macroscopic sector which resembles the Liquid Drop Mass (LDM) formula, and include deformation effects. HFB calculations are now able to fit known nuclear masses with deviations competitive with the '95 FRDM calculations, which are also being improved, while the most precise and robust nuclear mass predictions are given by the DZ model [4, 11].

The Liquid Drop Mass (LDM) formula captures the macroscopic features of the mass dependence on the number of neutrons N , of protons Z , and on its mass numbers $A = N + Z$.

It includes volume and surface terms, the Coulomb interaction between protons, Wigner and symmetry terms, linear and quadratic in the neutron excess $N - Z$, and a pairing term. It is generally assumed that the liquid-drop energy of a spherical nucleus is described by a Bethe-Weizsäcker mass formula [12], being a common practice to describe nuclear masses and radii of spherical closed-shell nuclei in terms of a mean field and add deformation and other shell effects as corrections [13].

It is the purpose of this contribution to show that, when nuclei with measured masses are grouped according with their quadrupole deformations, three Liquid Drop Mass formulas consistently allow for a fit with an RMS smaller than 750 keV for the set of most prolate deformed nuclei, while they are unable to fit the masses of spherical nuclei with an RMS smaller than 2000 keV. Semi-magic nuclei are fit with a similarly large RMS. The parameters of the three models are studied, showing that the surface symmetry term is the one which varies the most from one group of nuclei to another. In one model, isospin dependent terms are also found to exhibit strong changes.

Shell effects refer to the differences between the experimental binding energies [14] and the LDM predictions. In this work they are included employing linear, quadratic [15, 16], 3- and 4-body terms [13], functions of the number of valence nucleons, following the ideas of the Duflo-Zuker model [8, 9, 10]. The inclusion of shell effects allows for better fits, while the smallest rms are found again in the prolate deformed nuclei region.

2. The three Liquid Drop Mass formulas

Three Liquid Drop Mass formulas will be employed to analyze their ability to fit nuclear masses. One of them treats consistently volume and surface effects [13]. The second one incorporates explicitly isospin effects [12] while the third one includes quadratic isospin effects, the diffuseness correction to the Coulomb energy, the charge exchange correction term and the curvature energy [17].

In Ref. [13] an improved version of the liquid-drop mass formula with modified symmetry and Coulomb terms is built, following a consistent treatment of nuclear bulk and surface effects. The negative nuclear interaction energy is given by:

$$E_{LDM1} = -a_v A + a_s A^{2/3} + S_v \frac{4T(T+1)}{A(1+yA^{-1/3})} + a_c \frac{Z(Z-1)}{(1-\Lambda)A^{1/3}} - a_p \frac{\Delta}{A^{1/3}}, \quad (1)$$

where the pairing interaction is given by $\Delta = 2, 1$, and 0 for even-even, odd-mass and odd-odd nuclei, respectively. A modification Λ in the Coulomb term is included

$$\Lambda = \frac{N-Z}{6Z(1+y^{-1}A^{1/3})} - \frac{5\pi^2}{6} \frac{d^2}{r_0^2 A^{2/3}}, \quad (2)$$

Its presence in the denominator of the Coulomb term suggests that it can be viewed as a correction to the radius of the nucleus. The symmetry term employs $4T(T+1)$ instead of $(N-Z)^2$ to account for the Wigner energy. The Coulomb interaction is proportional to $Z(Z-1)$ to avoid the Coulomb interaction of a proton with itself.

The second LDM formula we are going to analyze was introduced in Ref. [12] by considering isospin effects. In this case the liquid-drop energy of a spherical nucleus is described by a modified Bethe-Weizsäcker mass formula

$$E_{LDM2} = -a_v A + a_s A^{2/3} + a_{sym} I^2 A + a_c \frac{Z(Z-1)}{A^{1/3}} (1 - Z^{2/3}) - a_{pair} \frac{\delta_{np}}{A^{1/3}}, \quad (3)$$

with isospin asymmetry $I = (N - Z)/A$. The pairing term is taken from [18]

$$\delta_{np} = \begin{cases} 2 - |I| & : N \text{ and } Z \text{ even} \\ |I| & : N \text{ and } Z \text{ odd} \\ 1 - |I| & : N \text{ even, } Z \text{ odd, and } N > Z \\ 1 - |I| & : N \text{ odd, } Z \text{ even, and } N < Z \\ 1 & : N \text{ even, } Z \text{ odd, and } N < Z \\ 1 & : N \text{ odd, } Z \text{ even, and } N > Z. \end{cases} \quad (4)$$

and the symmetry energy coefficient of finite nuclei is written as,

$$a_{sym} = c_{sym} \left[1 - \frac{\kappa}{A^{1/3}} + \frac{2 - |I|}{2 + |I| A} \right] \quad (5)$$

based on the conventional surface-symmetry term of liquid-drop model, with a small correction term for description of isospin dependence of a_{sym} . The symmetry energy coefficient a_{sym} increases with decreasing isospin asymmetry. This I correction term approximately describes the Wigner effect for heavy nuclei.

Different mass formulae derived from the liquid drop model and the pairing and shell energies of the Thomas-Fermi model have been studied and compared in Ref [17]. We selected for this study to include the diffuseness correction to the Coulomb energy, the charge exchange correction term and the curvature energy. In Ref. [17] it is reported that the Coulomb diffuseness correction Z^2/A term and the charge exchange correction $Z^{4/3}/A^{1/3}$ term play the main role to improve the accuracy of the mass formula. The Wigner term and the curvature energy can also be used separately but their coefficients are very unstable.

Their LDM formula is

$$E_{LDM3}(N, Z) = -a_v(1 - k_v I^2)A + a_s(1 - k_s I^2)A^{2/3} + a_k(1 - k_k I^2)A^{1/3} + \frac{3}{5} \frac{e^2 Z^2}{r_0 A^{1/3}} - f_p Z^2/A - a_{c,exc} Z^{4/3}/A^{1/3} + E_{pair} \quad (6)$$

The volume energy corresponding to the saturated exchange force and infinite nuclear matter is given by the first term. $I^2 A$ is the asymmetry energy of the Bethe-Weizsäcker mass formula. The second term is the surface energy. Its origin is the deficit of binding energy of the nucleons at the nuclear surface and corresponds to semi-infinite nuclear matter. The following term is the curvature energy. It results from non-uniform properties which correct the surface energy and depends on the mean local curvature. The decrease of binding energy due to the Coulomb repulsion is given by the fourth term, which has an adjustable charge radii $r_0 A^{1/3}$. The Z^2/A term is the diffuseness correction to the sharp radius Coulomb energy, also called also the proton form-factor. The $Z^{4/3}/A^{1/3}$ term is the charge exchange correction term. The pairing energies of the Thomas-Fermi model [19] were employed.

3. The fits

The coefficients of the three LDM were selected to minimize the root mean square deviation (RMS) when the predicted binding energies $BE_{th}(N, Z)$ are compared with the experimental ones $BE_{exp}(N, Z)$, reported in AME03 [14], modified so as to include more realistically the electron binding energies as explained in Appendix A of Lunney, Pearson and Thibault [4].

$$RMS = \left\{ \frac{\sum [BE_{exp}(N, Z) - BE_{th}(N, Z)]^2}{N_{nucl}} \right\}^{1/2}. \quad (7)$$

N_{nucl} is the number of nuclei in each group, listed in the fourth row of Table 3. The minimization procedure uses the routine Minuit [20].

The fits were performed employing the masses of nine groups of nuclei:

group	all	1	2	3	4	5	6	7	semi-magic
e_2 min	-0.65	-0.65	-0.11	0.00	0.04	0.12	0.18	0.23	
e_2 max	0.65	-0.11	0.00	0.04	0.12	0.18	0.23	0.65	
N_{nucl}	2149	258	252	332	272	307	364	364	185

Table 1. The nine groups on nuclei employed in the present study, their range of quadrupole deformation, and their number of nuclei.

- all nuclei whose measured masses are reported in AME03 [14], which have $N, Z \geq 8$,
- seven groups of nuclei whose quadrupole deformations, taken from the FRDM [21], lie in the ranges listed in the second and third row of Table 3,
- the group of all semi-magic nuclei, having $Z = 8, 20, 28, 50, 82$ or $N = 8, 20, 28, 50, 82$ or 126.

Notice that group 1 contains most of the oblate nuclei, that the more spherical nuclei belong to group 3, and that the more prolate deformed nuclei are included in groups 6 and 7.

For each LDM equation, nine fits were performed, one for each group of nuclei. In this way, nine sets of parameters were obtained, which minimize the RMS of each group of nuclei. Employing these nine sets of parameters, the RMS were estimated for all groups, whose RMS are shown in the next subsections.

3.1. Analysis of the LDM1 formula

Here we show the results obtained using Eq. (1) for the nine regions. In our calculations we select nine sets of fixed values for the parameters a_v, a_s, a_c, a_p, S_v and y . The results are exhibited in Tables 2 and 3.

Table 2. RMS (in keV) for the nine groups (columns), for the nine sets of parameters (rows), employing Eq. (1)

	all	1	2	3	4	5	6	7	semi-magic
set _{all}	2387	1842	3814	3080	2037	1495	1815	2060	3888
set ₁	3300	1313	3113	3420	3277	3173	5194	1476	4016
set ₂	4053	2917	1676	2433	3088	4273	6243	4701	2570
set ₃	3404	3313	2298	2063	2556	3408	4016	4721	2600
set ₄	2567	2431	3867	3096	1746	1741	1975	2626	3901
set ₅	2630	2097	4848	3696	2072	1053	1360	1735	4521
set ₆	2828	1942	5205	4262	2578	1469	870	1296	5078
set ₇	3169	1655	4077	4220	3543	2753	3643	746	5016
set _{semi}	4514	3816	1930	2665	3316	4777	6729	5306	2113

3.2. Analysis of the LDM2 formula

Here we show the results obtained using Eq. (3) for the nine regions. In our calculations we select nine sets of fixed values for the parameters $a_v, a_s, a_c, a_{pair}, c_{sym}$ and κ . The results are exhibited in Tables 4 and 5.

Table 3. Sets of parameters which minimize the RMS for the nine groups of nuclei, employing Eq. (1). In the last three rows the average value of each parameter, their dispersion and percentage variation $\%var = 100 \frac{\sigma}{|\text{average}|}$ are shown.

	a_v	a_s	a_c	a_p	S_v	y
set _{all}	15.822	18.491	0.703	6.048	30.701	2.616
set ₁	15.515	17.577	0.686	6.092	25.997	1.583
set ₂	15.883	18.622	0.712	5.733	28.711	2.284
set ₃	15.978	18.884	0.716	5.895	32.091	3.059
set ₄	15.879	18.564	0.708	4.103	31.459	2.666
set ₅	15.832	18.435	0.707	5.266	30.575	2.482
set ₆	15.609	17.835	0.689	5.334	29.762	2.388
set ₇	15.553	17.782	0.684	5.126	27.402	1.865
set _{semi}	15.775	18.012	0.712	5.500	26.407	1.489
average	15.761	18.245	0.702	5.455	29.234	2.270
σ	0.153	0.426	0.012	0.579	2.093	0.495
% var	1.0	2.3	1.7	10.6	7.1	21.8

Table 4. RMS (in keV) for the nine groups (columns), for the nine sets of parameters (rows), employing Eq. (3)

	all	1	2	3	4	5	6	7	semi-magic
set _{all}	2374	1788	3733	3081	2044	1502	1855	2058	3916
set ₁	3208	1254	3114	3400	3243	3077	4947	1435	4060
set ₂	4092	2965	1675	2380	3027	4318	6393	4697	2380
set ₃	3475	3352	2173	2069	2596	3518	4183	4816	2551
set ₄	2561	2408	3761	3052	1762	1771	2036	2688	3897
set ₅	2616	2073	4823	3702	2083	1021	1311	1704	4548
set ₆	2795	1865	5130	4234	2619	1435	838	1242	5090
set ₇	3074	1589	4216	4238	3541	2590	3138	656	5093
set _{semi}	4677	3915	1959	2722	3427	4953	7036	5466	2056

3.3. Analysis of the LDM3 formula

Here we show the results obtained using Eq. (6) for the nine regions. In our calculations we select nine sets of fixed values for the parameters a_v , a_s , r_0 , a_{pair} , k_v , k_s , f_p , $a_{c,exc}$, a_k and k_k . The results are exhibited in Tables 6 and 7.

3.4. LDM and deformation

Tables 2, 4, 6 display the RMS obtained with the three LDM formulas. Each row refers to one fixed set of parameters, each column to one group of nuclei.

For the LDM1, Table 2, the global RMS is 2.39 MeV for the 2149 nuclei. In some groups, containing around two or three hundred nuclei, the RMS obtained with this set of parameters is smaller than the global one. The largest RMS are found in groups 2 and 3, those containing spherical nuclei. Along each column, corresponding to one group of nuclei, the smallest RMS

Table 5. Sets of parameters which minimize the RMS for the nine groups of nuclei, employing Eq. (3). In the last three rows the average value of each parameter, their dispersion and percentage variation $\%var = 100 \frac{\sigma}{\text{average}}$ are shown.

	a_v	a_s	a_c	a_{pair}	c_{sym}	κ
set _{all}	15.711	18.920	0.720	6.989	30.045	1.587
set ₁	15.432	18.119	0.700	6.793	26.488	1.260
set ₂	15.695	18.802	0.722	7.443	27.979	1.441
set ₃	15.903	19.395	0.737	6.717	30.582	1.642
set ₄	15.729	18.877	0.721	4.685	30.858	1.635
set ₅	15.685	18.756	0.721	6.210	29.969	1.548
set ₆	15.493	18.254	0.703	6.276	29.490	1.553
set ₇	15.500	18.398	0.701	6.193	27.715	1.361
set _{semi}	15.627	18.329	0.724	5.327	26.043	1.099
average	15.642	18.650	0.717	6.293	28.797	1.458
σ	0.138	0.383	0.012	0.801	1.687	0.175
% var	0.9	2.0	1.7	12.7	5.8	12.0

Table 6. RMS (in keV) for the nine groups (columns), for the nine sets of parameters (rows), employing Eq. (6)

	all	1	2	3	4	5	6	7	semi-magic
set _{all}	2422	1884	3773	3186	2031	1519	1861	2125	3937
set ₁	3576	1183	3067	3785	3461	3456	5854	1283	4424
set ₂	4211	3032	1597	2648	2928	4409	6596	4871	2333
set ₃	3563	3398	2197	2151	2618	3628	4455	4808	2499
set ₄	3049	3035	3849	4090	1517	1764	2560	3456	3952
set ₅	2776	2406	4838	4004	1927	986	1587	2110	4464
set ₆	2895	2083	5073	4459	2701	1471	819	1579	5103
set ₇	3200	1484	4128	4317	3506	2701	3790	629	5135
set _{semi}	4707	3776	1781	2720	3337	4962	7321	5392	1967

always corresponds the set of parameters obtained fitting in this group, as expected. These RMS values are displayed in bold numbers. It is remarkable that the smallest RMS values are found in the two groups having the more oblate deformed nuclei, in groups 6 and 7, with 0.87 and 0.75 MeV, respectively. On the other hand, the best fit of the spherical nuclei in group 3 has an RMS larger than 2.0 MeV, and those nuclei with very small quadrupole deformation, belonging to groups 2 and 4, have RMS larger than 1.6 MeV. Consistently, the semi-magic group of nuclei, which have small quadrupole deformations, have an RMS of 2.1 MeV.

By analyzing the RMS for each column, it is possible to notice that the smaller RMS are always found around the smallest one. It supports the idea that the division in the seven groups with different deformations makes sense, because the parameters obtained fitting nuclei with a close deformation produce also a small RMS. Notice, for example, that among the RMS of groups 1 and 7, the more oblate and prolate deformed, respectively, the largest RMS are found with the sets of parameters 2 and 3, i.e. those fitted for spherical nuclei. On the opposite

Table 7. Sets of parameters which minimize the RMS for the nine groups of nuclei, employing Eq. (6). In the last three rows the average value of each parameter, their dispersion and percentage variation $\%var = 100 \frac{\sigma}{|\text{average}|}$ are shown.

	a_v	a_s	r_0	a_{pair}	k_v	k_s	f_p	$a_{c,exc}$	a_k	k_k
set _{all}	15.647	20.610	1.198	-1.002	2.079	4.451	1.923	0.296	-4.738	30.717
set ₁	15.834	21.074	1.203	-0.905	1.305	-0.812	1.265	0.304	-5.910	-12.666
set ₂	15.557	20.860	1.213	-1.037	1.954	4.339	2.583	0.418	-4.719	32.562
set ₃	15.673	19.718	1.234	-1.005	1.734	2.072	2.071	0.394	-1.248	21.077
set ₄	15.671	22.448	1.227	-0.726	2.720	9.059	2.936	0.661	-7.069	58.552
set ₅	15.577	21.060	1.196	-0.873	2.398	6.963	2.259	0.334	-6.359	44.965
set ₆	15.611	20.997	1.158	-0.963	2.403	6.781	1.457	0.084	-8.514	33.205
set ₇	15.806	20.834	1.204	-0.962	1.393	-0.390	1.393	0.289	-4.988	-12.233
set _{semi}	15.363	18.242	1.221	-0.904	1.418	0.666	2.577	0.211	0.538	-0.408
average	15.638	20.649	1.206	-0.931	1.934	3.681	2.052	0.332	-4.779	21.752
σ	0.132	1.077	0.021	0.089	0.479	3.313	0.558	0.149	2.662	23.694
% var	0.8	5.2	1.7	9.5	24.8	90.0	27.2	44.9	55.7	108.7

side, along columns 3 and 4, containing the more spherical nuclei, the largest RMS are found employing the sets 1 and 7, and in some cases 2 and 6.

It is tempting to conclude that the LDM is best suited for the description of prolate quadrupole deformed nuclei. In order to find support for this conclusion, it is worth to analyze the results presented in Tables 4 and 6. The global RMS for LDM2 is 2.37 MeV, and for LDM3 is 2.42 MeV. The three fits are pretty close to each other. The smallest RMS are always found for the 364 nuclei belonging to group 7, the more prolate deformed, with 0.65 MeV and 0.63 MeV for LDM2 and LDM3, respectively. The masses of the 332 spherical nuclei included in group 3, and the 185 semi-magic nuclei can hardly be fitted with and RMS smaller than 2.0 MeV. Also the correlations between groups are similar for these two other models.

This is the most relevant result reported in this contribution: **the Liquid Drop Model is best suited to describe the masses of prolate deformed nuclei than of spherical nuclei.**

3.5. Comparison of the three LDM formulas

While being close to each other, the three LDM formulas employed in this work have differences which are worth to be studied in detail. A statistical analysis of the parameters of each model probed to be useful in previous studies [11]. From the nine values of each parameter, their average, fluctuations and percentage fluctuations are presented in the last three rows of Tables 3, 5 and 7.

The LDM1, Eq. (1) and LDM2, Eq. (3) are very similar. Their volume, surface and Coulomb parameters, listed in Tables 3 and 5, have nearly equal numerical values, and are all of them very stable, with a dispersion along the nine sets smaller than 2%. The pairing parameter fluctuate between 10% and 13%, and the coefficient of the symmetry term around 6-7%. The surface symmetry term y in LDM1 is the most unstable, with fluctuations of 22%, while its counterpart in LDM2, κ , has fluctuations of 12%. In this subtle sense, LDM2 could be considered more stable than LDM1. It could be useful to perform a deeper comparison of the surface symmetry terms, which in one model was obtained asking for consistency between the volume and surface

contributions, while in the other was designed to incorporate the isospin dependence explicitly.

The LDM3, Eq. (6), has also stable volume and Coulomb (charge radius) terms, but the surface term has fluctuations of the order of 5%, as seen in Table 7. The pairing parameter, as in the other two models, fluctuates around 10%. On the other hand, the model has ten parameters, and the remaining six have enormous fluctuations in both magnitude and sign. These instabilities of the model parameters could be interpreted as a weakness which should be addressed.

4. Shell effects

The main obstacle for an accurate description of spherical nuclei employing a Liquid Drop Mass formula are the shell effects around closed shells.

In the literature many different ways of implementing shell corrections to the LDM can be found; in general, these methods are rather laborious. A simple method was proposed in Refs. [15, 16] based on counting the number of valence nucleons. This shell correction is linear and quadratic in the total number of valence nucleons n and z ,

$$E_{LDMM} = E_{LDM1} + b_1(n + z) + b_2(n + z)^2 \quad (8)$$

where n and z are the numbers of valence neutrons and protons (particle- or hole-like) and b_i are parameters. Inclusion of these two terms in the LDM mass formula (1) reduces the rms deviation from 2.39 to 1.05 MeV.

Following Ref. [13], we employ also an upgraded version of the terms (8), which is suggested by the microscopic mass formula of Duflo and Zuker [10, 22]:

$$E_{LDMM'} = E_{LDMM} + a_1 S_2 + a_2 (S_2)^2 + a_3 S_3 + a_{np} S_{np} \quad (9)$$

where

$$S_2 = \frac{n\bar{n}}{D_n} + \frac{z\bar{z}}{D_z}, \quad S_3 = \frac{n\bar{n}(n - \bar{n})}{D_n} + \frac{z\bar{z}(z - \bar{z})}{D_z}, \quad S_{np} = \frac{n\bar{n}}{D_n} \frac{z\bar{z}}{D_z}, \quad (10)$$

with $\bar{n} = D_n - n$ and $\bar{z} = D_z - z$, where $D_n(D_z)$ is the degeneracy of the neutron (proton) valence shell.

They include 2-, 3- and 4-body terms. The quadratic term is associated to configuration mixing and the cubic one to a genuine three body force [18].

In Table 8 the RMS of the best fits for all the nuclei, for the seven sets of nuclei grouped according to their deformations, and for the semi-magic nuclei, are presented for the Liquid Drop Model, Eq. (1), for the Modified Liquid Drop Model, Eq. (8) and for the model including 3- and 4-body terms, Eq. (9). The first row corresponds to the RMS listed in Table 2 in bold face numbers. It is clear that the inclusion of microscopic terms improves the fits. The global RMS, for all nuclei, diminished from its LDM value of 2.39 MeV to 1.07 MeV and 0.89 MeV. The most impressive reductions in the RMS are found in the spherical nuclei grouped in region 3, which drops from 2.06 MeV to 1.01 MeV and 0.90 MeV, and for the semi-magic nuclei, whose RMS diminishes from 2.11 MeV to 1.04 MeV and 0.82 MeV.

On the other hand, the nuclei in region 7, the most prolate deformed, are the best fitted in the LDMM, while the spherical nuclei in region 3 and the semi-magic nuclei have the largest RMS. The inclusion of 2-, 3- and 4-body terms in LDMM' seems to succeed in introducing deformation effects. Regions 1 and 4 to 7 have all RMS between 562 and 623 keV. Spherical and semi-magic nuclei remain to be those with the largest RMS.

Table 8. RMS (in keV) of the best fit for each of the nine groups (columns), employing Eqs. (1), (8) and (9), respectively.

	all	1	2	3	4	5	6	7	semi-magic
LDM1	2387	1313	1676	2063	1746	1053	869	746	2113
LDMM	1075	797	962	1007	828	711	792	616	1038
LDMM'	888	623	741	902	634	562	620	575	817

5. Conclusions

Along this contribution we have shown that the Liquid Drop Model is best suited to describe the masses of prolate deformed nuclei than of spherical nuclei. The analysis was performed employing three different Liquid Drop Mass formulas. With them, the nuclear masses nuclei grouped in eight sets with similar quadrupole deformations were fitted. For the three LDM models it was found that the masses of prolate deformed nuclei can be described with remarkable precision for a LDM, with an RMS smaller than 750 keV, while the masses of spherical and semi-magic nuclei are those worst described, with RMS larger than 2000 keV.

The dispersion of the parameters of the three models were studied comparing the fits for the different groups of nuclei. We found that in the three the surface symmetry term is the one which varies the most from one group of nuclei to another. In the model of Ref. [17], isospin dependent terms were found to exhibit strong changes, making this model the least robust of the three under this criterion.

The inclusion of shell effects allows for better fits, which continue to be better in the prolate deformed nuclei region. The Duflo-Zuker model is based in a microscopic description of shell effects, and describes deformation through a change in valence occupations. It remains a challenge to see if the DZ mechanism to incorporate deformation effects can be successfully employed by other models.

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References

- [1] A. Bohr and B.R. Mottelson, *Nuclear Structure* v. I, (World Scientific, Singapore, 1998).
- [2] C.E. Rolfs and W.S. Rodney, *Cauldrons in the Cosmos* (University of Chicago Press, Chicago, 1988).
- [3] Klaus Blaum, Phys. Rep. 425 (2006) 1.
- [4] D. Lunney, J.M. Pearson, C. Thibault, Rev. Mod. Phys. 75 (2003) 1021.
- [5] P. Möller, J.R. Nix, W.D. Myers, W.J. Swiatecki, Nuclear ground-state masses and deformations. At. Data Nucl. Data Tables 59 (1995) 185.
- [6] S. Goriely, F. Tondeur, J.M. Pearson, A Hartree-Fock nuclear mass table. Atom. Data Nucl. Data Tables 77 (2001) 311; S. Goriely, M. Samyn, J.M. Pearson, Phys. Rev. C 75 (2007) 064312.
- [7] S. Goriely, N. Chamel, J.M. Pearson, Phys. Rev. Lett. **102** (2009) 152503; S. Goriely, S. Hilaire, M. Girod, and S. Péru, Phys. Rev. Lett. **102** (2009) 242501.
- [8] J. Duflo, Nucl. Phys. A 576 (1994) 29.
- [9] A.P. Zuker, Nucl. Phys. A 576 (1994) 65.
- [10] J. Duflo and A.P. Zuker, Phys. Rev. C 52 R23 (1995) R23.
- [11] J. Mendoza-Temis, I. Morales, J. Barea, A. Frank, J.G. Hirsch, J.C. López-Vieyra, P. Van Isacker and V. Velázquez, Nucl. Phys. A **812** (2008) 28.
- [12] N. Wang, M. Liu and X. Wu, Phys. Rev. C **81** 044322 (2010).
- [13] A.E.L. Dieperink, P. Van Isacker, Eur. Phys. J. A 42 (2009) 269.
- [14] G. Audi, A.H. Wapstra, C. Thibault, Nucl. Phys. A 729 (2003) 337.

- [15] A.E.L. Dieperink, P. Van Isacker, Eur. Phys. J. A 32 (2007) 11.
- [16] J. Mendoza-Temis, A. Frank, J.G. Hirsch, J.C. López Vieyra, I. Morales, J. Barea, P. Van Isacker, V. Velázquez, Nucl. Phys. A 799 (2008) 84.
- [17] G. Royer, M. Guilbaud and A. Onillon, Nucl. Phys. **A 847** (2010) 24.
- [18] J. Mendoza-Temis, J.G. Hirsch, A.P. Zuker, Nucl. Phys. **A 843** (2010) 14.
- [19] W.D. Myers, W.J. Swiatecki, Nucl. Phys. **A 601** (1996) 141.
- [20] F. James, Minuit: Function Minimization and Error Analysis Reference Manual, Version 94.1, CERN (1994); <http://wwwasdoc.web.cern.ch/wwwasdoc/minuit/minmain.html>
- [21] P. Möller, J. R. Nix, W. D. Myers, and W. J. Swiatecki, Atomic Data Nucl. Data Tables **59** (1995) 185.
- [22] A.P. Zuker, Rev. Mex. Fis. S **54** (2008) 129, rmf.fciencias.unam.mx/pdf/rmf-s/54/3/54_3_129.pdf.